Kinematics Unit

MOTION IN TWO DIMENSIONS

IMPLE KINEMATICS

emember the equations of motion from grade 11:

$V_o = V_i = V_{initial}$

$= v_{av}t \qquad v_f = v_o + at \qquad d = v_o t + at^2 \qquad v_f^2 = v_o^2 + 2ad \qquad v_{av} = \dfrac{v_f + v_o}{2}$

s before, these formulas can be used to solve a variety of motion problems, particularly where elocity, distance and time are involved. Recall too, that acceleration is a **vector** quantity and direction nust be considered when dealing with such problems. For example, if any object is projected into the ir, it is under a constant acceleration of **9.8 m/s^2** *downwards*.

luch of the study of kinematics in Physics 12 involves analysis of objects launched into the air (called rojectile motion). In these cases, the only acceleration considered is that of gravity, which acts ertically downward, regardless of the direction of the object's motion.

d = distance (scalar)
$\vec{d}$ = displacement (vector)

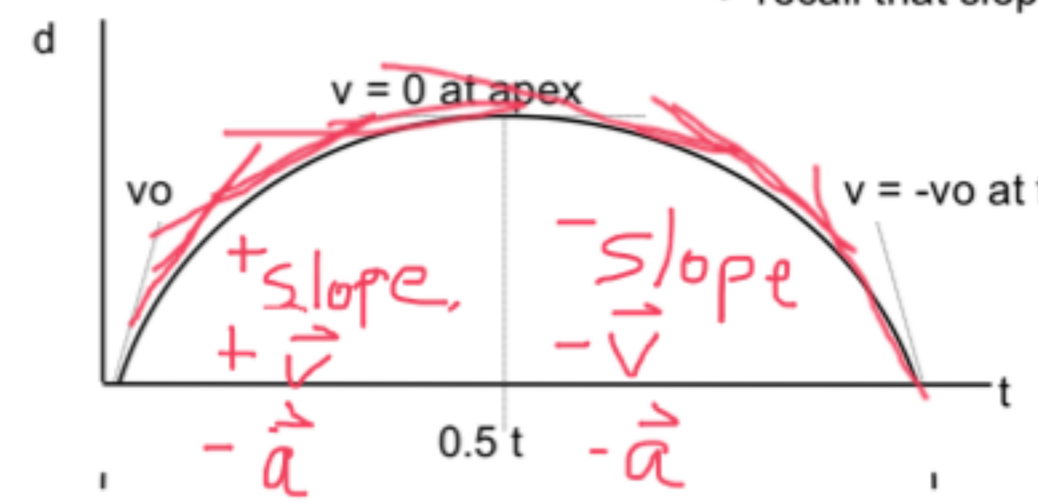
☆ $\vec{a}$ = acceler
$\vec{v}$ = velocity
v = speed

EX) Start by considering the simple motion of a baseball tossed straight upwards from ground level and allowed to drop back down to that level where it is caught (similar to Example 1).

a) Examine a distance-time graph of the baseball's motion:

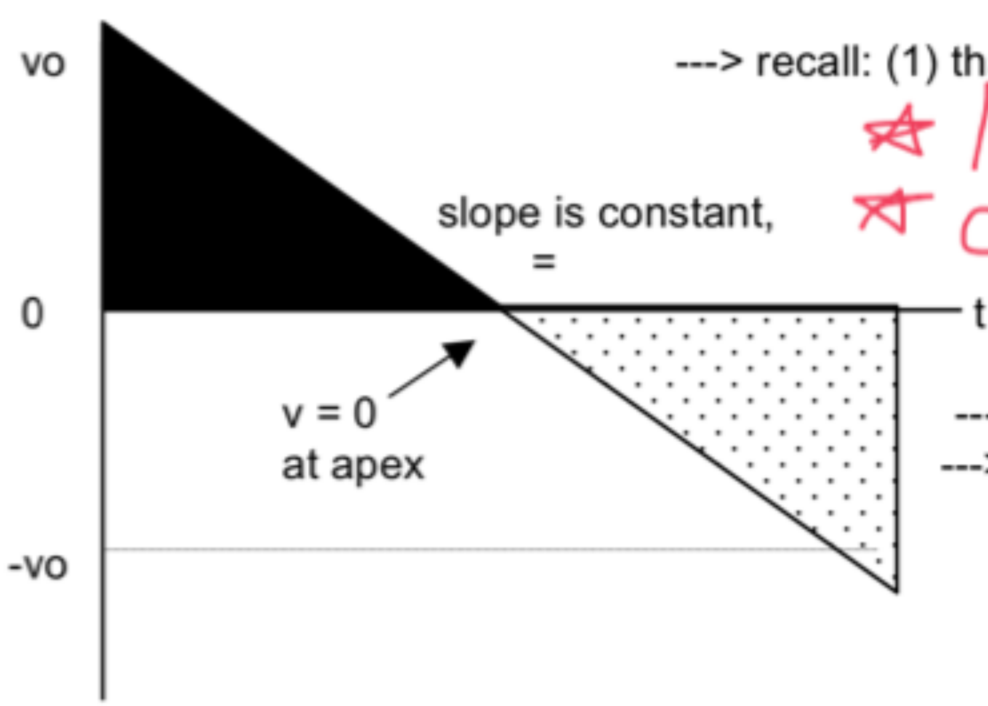
---> recall that slope = velocity (use tangent lines here)

= rise / run



☆ curved line = non-linear velocity = acceleration is occurring

b) Now examine a velocity-time graph of the baseball's motion:



---> recall: (1) that slope = acceleration

☆ linear line = constant $\vec{a}$

☆ curved line = non-uniform ($\vec{a}$ is changing instantaneously)

---> recall: (2) that area = displacement

---> describe the meaning of the two areas shown:

Note: analysis of graphs is an important aspect of this course. For any given graphing relationship, be sure that you can:

from Pre-Calc

a) make an equation from the line
b) describe the significance of the slope of the line
c) state the meaning of the area under the graph.

Ex.#1. A rocket is projected upwards at an initial velocity of 750 m/s.
If there is no air friction,
(a) how long does the rocket take to reach its highest point?
(b) how high does it go?
(c) if it lands at the same level as launch, how long is the rocket in the air?
(76.5 s, 2.87×10^4 m, 153 s)

a) @ highest point
$V_f = 0$ m/s

V_i ✓
V_f ✓
t
a ✓
d

$V_f = V_i + at$
$0 = 750 + (-9.8)t$
$9.8t = 750$
$\boxed{t = 76.5 s}$

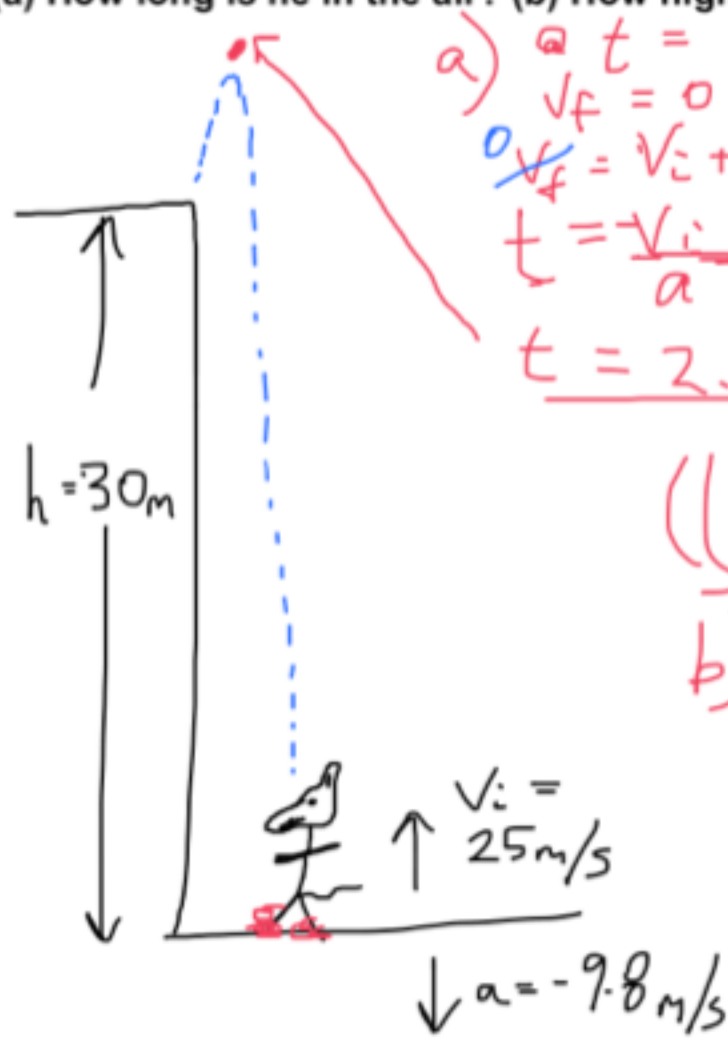
b) $d = V_o t + \frac{1}{2}at^2$
$= (750)(76.5) + .5(-9.8)(76.5)^2$
$d = 57375 - 28676$
$\boxed{d = 2.87 \times 10^4 m}$

c) • both t's are equal
• both V_o and V_f are equal but with different directions

t_{up} t_{down}

$\boxed{t_f = t_{up} + t_{down} = 153 s}$

Ex #2. Wile E. Coyote wants to jump onto a cliff 30.0 meters high so, using his Acme spring-loaded tennis shoes, he jumps straight upwards at 25.0 m/s and safely lands on the cliff edge.
(a) How long is he in the air? (b) How high did he actually jump? (3.17 s, 32 m)



a) @ t = ?
$V_f = 0$
$\cancel{V_f} = V_i + at$
$t = \frac{-V_i}{a} = \frac{-25}{-9.8}$
$t = 2.55 s$ to apex

b) $d = V_o t + \frac{1}{2}at^2$
$= 25(2.55) + 0.5(-9.8)(2.55)^2$
$= 63.75 - 31.9$
$\boxed{d = 31.9 m}$

~~time~~
$V_o = 0$
$V_f = ?$
$a = -9.8 m/s^2$
$d = 2$
$t = ?$

$d = \cancel{V_o t} + \frac{1}{2}at^2$
$-2 = \frac{1}{2}(-9.8)t^2$
$t^2 = \frac{-4}{-9.8}$
$t = \sqrt{0.41}$
$t = 0.64 s$

$t_{total} = t_{up} + t_{down}$
$t = 2.55 + 0.64$
$\boxed{t = 3.18 s}$

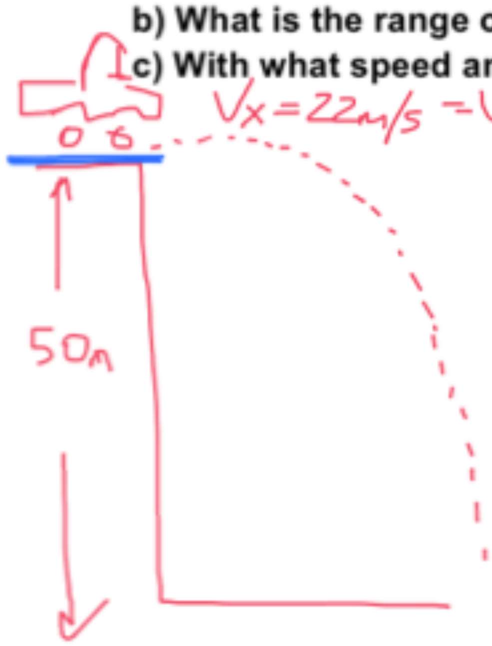
HORIZONTAL PROJECTILES

Now we examine a two-dimensional motion problem taught in Physics 11, where a projectile is launched horizontally from a point above the ground.

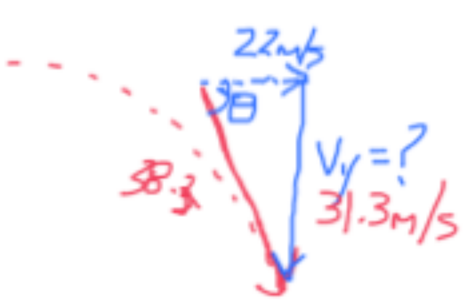
Recall the following: $\vec{a}$ only in y-dir

- horizontal motion is unrelated to vertical motion
- gravity acts only in the vertical direction; ∴ horizontal velocity must be *constant*
- can determine horizontal displacement using **d = v_{av}t**
- can use kinematics formulas when dealing with vertical displacement
- Hor/Vert tables

Ex. #3. Consider a car going 22.0 m/s that drives off a 50.0 m-high cliff.
a) How long does the car take to hit the ground? (3.16 s)
b) What is the range of the car? (69.5 m) (x-distance)
c) With what speed and direction does it hit? (38.2 m/s at 55° down)



vert (y)	hor (x)
$V_i = 0$	$V_i = V_f = 22$ m/s
$a = -9.8$ m/s^2	$t_{vert} = t_{hor} = 3.19$ s
$d = -50$ m	$d_x = V \cdot t$
$t = ?$	$= 22(3.19)$
$d = \cancel{V_0 t} + \frac{1}{2}at^2$	b) $\boxed{d_x = 70.2\text{ m}}$
$t = \sqrt{\frac{2d}{a}}$	
$= \sqrt{\frac{2(-50)}{-9.8}}$	
a) $\boxed{t = 3.19\text{ s}}$	



need V_f in y-dir

$V_f = \cancel{V_0} + at$
$= (-9.8)(3.19)$
$= -31.3$ m/s (down)

mag

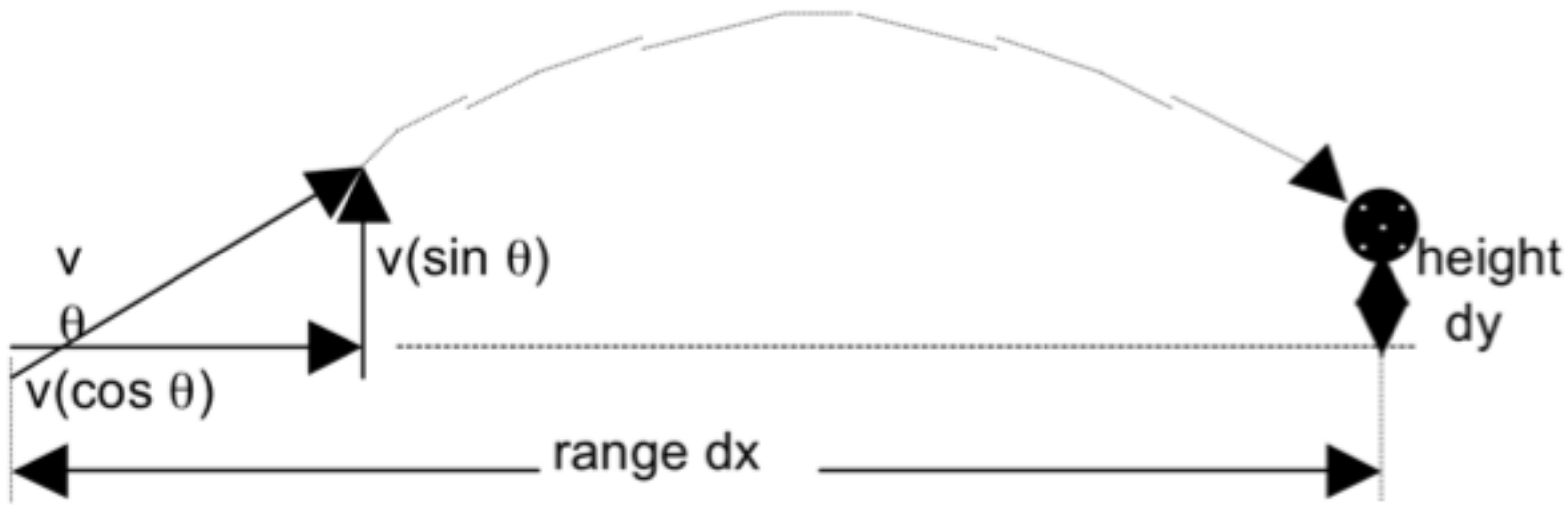
$a^2 + b^2 = c^2$
$c = \sqrt{22^2 + (31.3)^2}$
$c = 38.3$ m/s

dir

$\tan\theta = \frac{o}{a}$
$\theta = \tan^{-1}\left(\frac{31.3}{22}\right) = 55°$ S of E

c) $\boxed{38.3\text{ m/s @ }55°\text{ S of E}}$

First, we must resolve the given initial velocity into horizontal and vertical components by trig:



Next, arrange all information into a horizontal and vertical list (as before):

Hor.	Vert.
vconst = v cosθ	vo = v sinθ
a = 0	**a =-9.8m/s2**
	(up is positive)

Using this information, we can:

---> use vert. data to find total time up and down

 ---> use the total time to find overall range

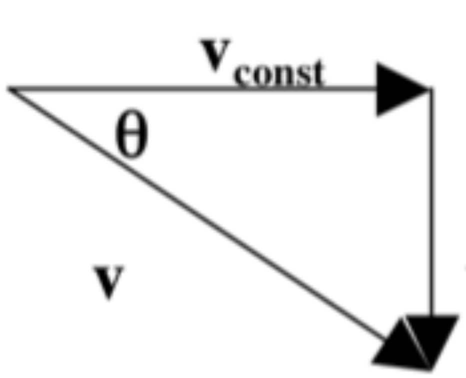
---> find both range and height, given any time **t**

---> find the final resultant velocity, given any time **t:**

 ---> remember that horizontal speed is constant

 ---> kinematics can be used to find the final vertical speed

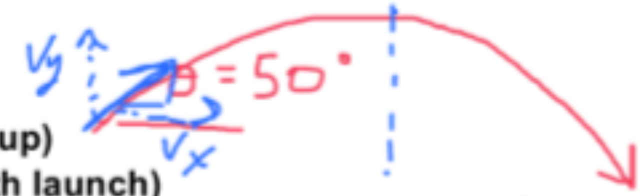
 ---> finally, vector-add these velocity components to find the resultant speed and direction:



$$\mathbf{v_y = v_o + at} \qquad \mathbf{v = \left(v_{const}^2 + v_y^2\right)^{\frac{1}{2}}} \qquad \mathbf{\theta = \tan^{-1}\left(\frac{v_y}{v_{const}}\right)}$$

Ex. #5. An artillery shell is fired over level ground, at 400. m/s and at an angle of 50° to the horizontal. Find:

(a) total time in the air. (62.6 s)
(b) how high the shell rises. (4800 m)
(c) the location of the shell after 25.0 seconds. (7900 m, 35.5° up)
(d) the velocity at impact. (400 m/s, 50° down ---> compare with launch)
(e) the range of the shell. (16 000 m)



x	y
$V_x = V\cos\theta$	$V_y = V\sin\theta$
$= 400\cos 50$	$= 400\sin 50$
$V_x = 257\,m/s$	$V_y = 306\,m/s$

a) t_1 t_2 $\quad t_T = t_1 + t_2$

$\cancel{V_f} = V_0 + \cancel{a}t$ ($V_f = 0$)

$0 = 306 - 9.8t$

$t = \frac{306}{9.8}$

$= 31.2\,s$ up

therefore $\therefore 2t =$ total time

$\boxed{t = 62.4\,s}$

b) $d = V_0 t + \frac{1}{2}at^2$

$= 306(31.2) + 0.5(-9.8)(31.2)^2$

$\boxed{d = 4770\,m}$

c) $d_x = V_x t$
$= (257)(25)$
$= 6425\,m$

$d_y = V_0 t + \frac{1}{2}at^2$
$= 306(25) + 0.5(-9.8)(25)^2$
$= 4587.5\,m$

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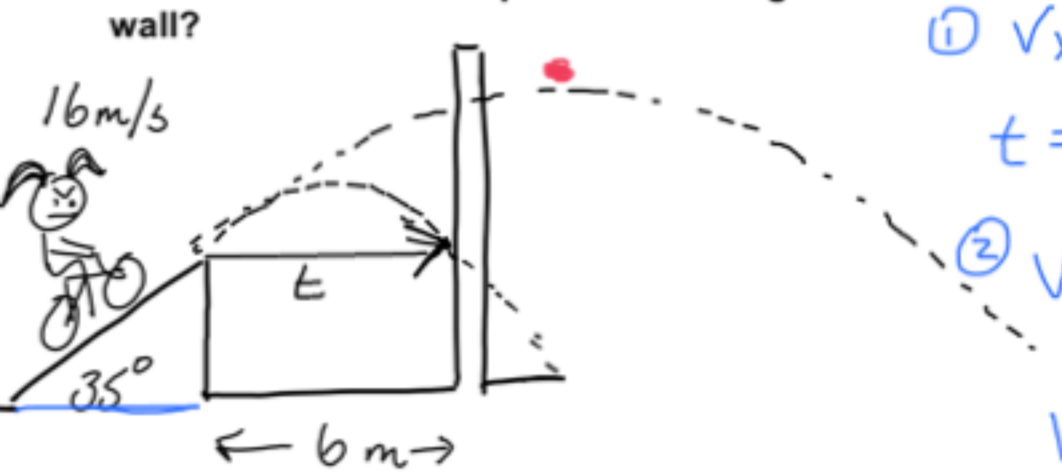
Ex. #6. A doofus attempts the following stunt. What is the final velocity when she hits the wall?



① V_x = cnst ∴ use to find t

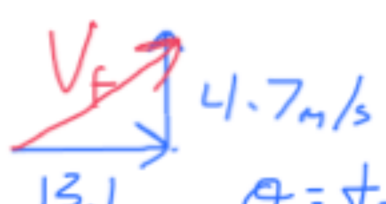
$t = \frac{d}{v} = \frac{6}{13.1} = 0.46\,s$

② $V_y = V_0 + at$
$= 9.2 - 9.8(0.46)$
$V_y = 4.69\,m/s$

$V_x = 16\cos 35 = 13.1\,m/s$

$V_y = 16\sin 35 = 9.2\,m/s$

@ wall

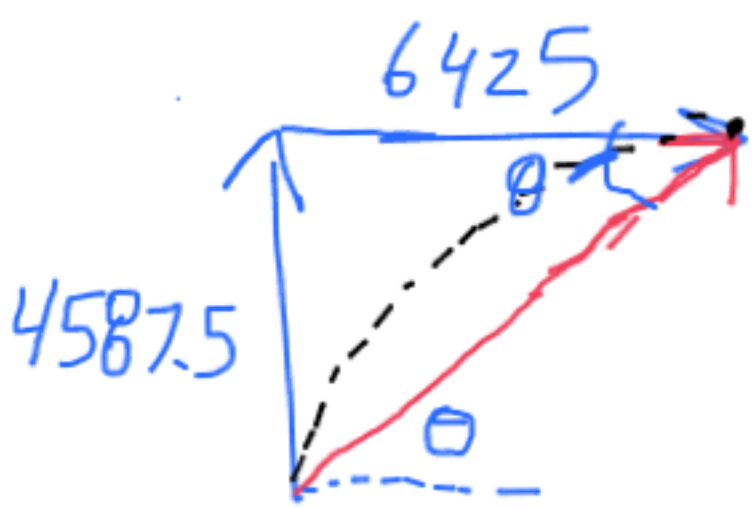


$V_f^2 = 4.7^2 + 13.1^2$

$\boxed{V_f = 13.9\,m/s}$ @

$\theta = \tan^{-1}\left(\frac{4.69}{13.1}\right) = \boxed{19.7°\ N\ of\ E}$

5c) cont....



$4587.5^2 + 6425^2 = \vec{d}^{\,2}$

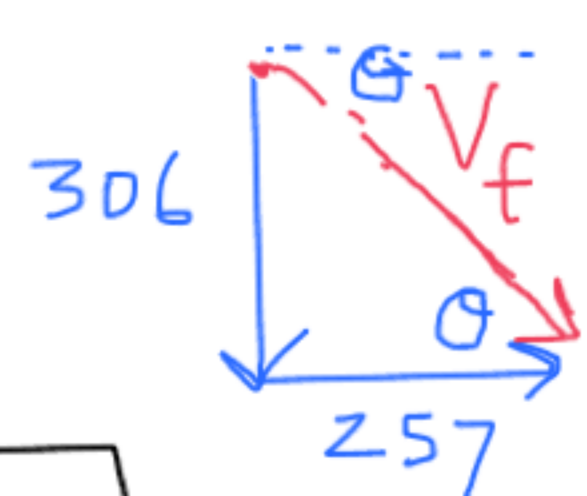
$\boxed{d = 7894\,m \text{ @ } 35.5^\circ \text{ N of E}}$

$\theta = \tan^{-1}\left(\frac{4587.5}{6425}\right) = 35.5^\circ$

d) $V_{fy} = V_0 + at$

$= 0 - 9.8(31.2)$

$V_{fy} = -306\,m/s$



$V_f^2 = 306^2 + 257^2$

$V_f = 400\,m/s$

$\theta = \tan^{-1}\left(\frac{306}{257}\right)$

$\theta = 50^\circ$

$\boxed{V_f = 400\,m/s \text{ @ } 50^\circ \text{ S of E}}$

e) $d_x = V_x t$

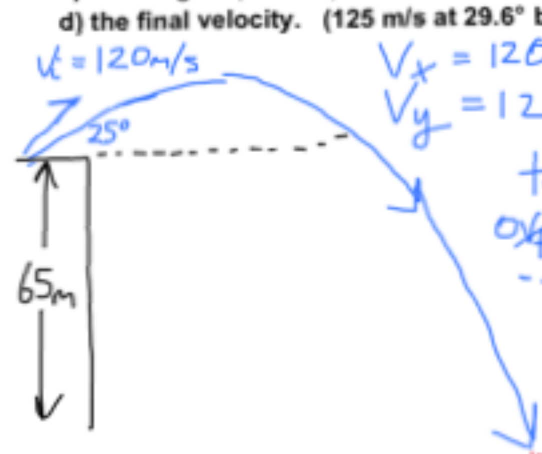
$= 257(62.4)$

$\boxed{d_x = 16037\,m \text{ E}}$

Ex. #7. The case of a projectile launched at an angle to the horizontal from a point above the ground.

In this example the mass is shot at 25° above the horizontal at a velocity of 120 m/s from a height of 65.0 meters. Find:

a) the time in the air. (11.5 s)
b) highest height of the projectile above the ground. (196 m)
c) the range. (1250 m)
d) the final velocity. (125 m/s at 29.6° below horizontal)



$V_x = 120\cos 25 = 109 m/s$

$V_y = 120\sin 25 = 51 m/s$

t_{up}

$\cancel{V_f}^{0} = V_0 + at$

$-t = \frac{V_0}{a}$

$t_{up} = \frac{50.7}{9.8} = 5.2 s$

t_{down}

find V_{fy}

$V_f^2 = V_0^2 + 2ad$

$= -51^2 \cdot 2(-9.8)(65)$

$V_f^2 = 3844/s$

$V_{fy} = 62 m/s \downarrow$

$V_f = V_0 + at$

$-62 = 0 - 9.8t$

$t = \frac{62}{9.8}$

$t_{dn} = 6.3 s$

time total $= 5.2 + 6.3 = \boxed{11.5 s}$

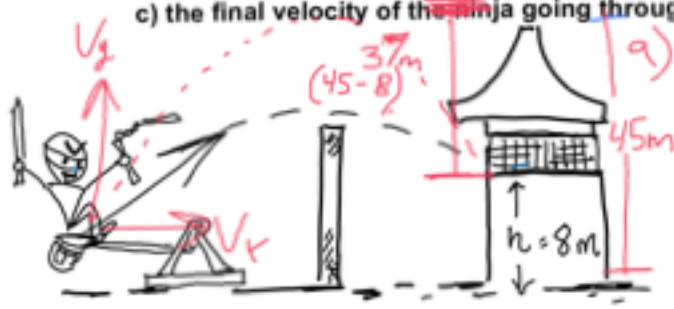
b) $d = V_0 t + \frac{1}{2}at^2$

$= 50.7(5.2) + 0.5(-9.8)(5.2)^2$

$d = 131 m + 65 m = \boxed{d = 196 m}$

cont. on next pg →

Ex. #8. Examine the scenario of the daring ninja with flair attempting to catapult himself over a wall and into a temple window 8m above the ground.
If the catapult can launch the ninja at 56 m/s at an angle of 32°, find:
a) the distance the catapult should be positioned ( )
b) maximum height of the ninja ( )
c) the final velocity of the ninja going through the window



a) find time using V_y
i) max height
ii) 8m height going down

i) $\cancel{V_f}^{0} = V_i + at$

$t = \frac{29.7}{9.8} = 3 sec$ up

ii) $d = \cancel{V_0 t}^{0} + \frac{1}{2}at$

$-37 = -0.5(9.8)t^2$

$t^2 = 7.6$

$t = 2.74 sec$

$t_t = t_{up} + t_{down}$

$= 3 + 2.74$

$t = 5.74 s$

$V_x = \frac{d_x}{t}$

$d_x = V_x t$

$= (47.5)(5.74)$

$\boxed{d_x = 272.7 m \text{ from the temple}}$

$V_x = 56\cos 32 = 47.5 m/s$

$V_y = 56\sin 32 = 29.7 m/s$

b) max height @ t = 3 sec

$d = V_i t + \frac{1}{2}at^2$

$= 29.7(3) - 0.5(9.8)(3)^2$

$\boxed{d_{max} = 45 m}$

c) $V_x = 47.5 m/s$

$V_y = ?$

$V_f = V_0 + at$

$= 0 - 9.8(2.74)$

$V_f = -27 m/s$ (down)

$V_f = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{47.5^2 + 27^2}$

$\theta = \tan^{-1}\left(\frac{27}{47.5}\right)$

$= 29.6°$

$\boxed{V_f = 54.6 m/s @ 29.6° S of E}$

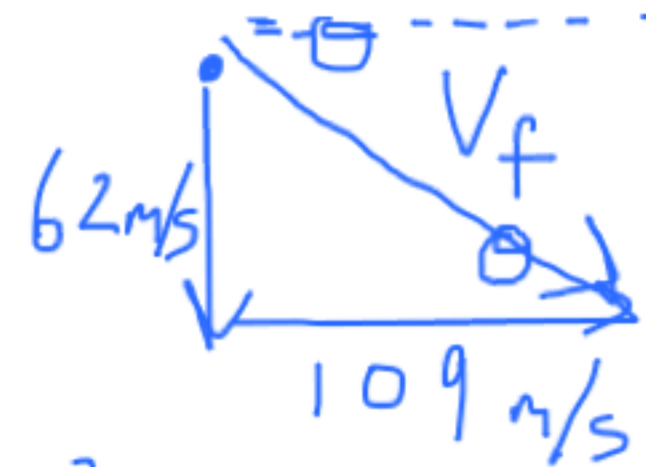
c) Q7

$d_x = V_x t$

$\quad = 109(11.5)$

$\boxed{d_x = +1253\ m}$

d)



$V_f^2 = 62^2 + 109^2$

$\boxed{V_f = 125\ m/s\ @\ 29.6^\circ\ S\ of\ E}$

$\tan\theta = \frac{o}{a}$

$\theta = \tan^{-1}\left(\frac{62}{109}\right) = 29.6^\circ$

HOR. PROJ. USING QUADRATICS

Ex. #9. A foul-mouthed Troll and an eclectic Weirdo are diagramed in the scenario below. If the Weirdo throws a pie with a velocity of 65 m/s at an angle of 37° find:

- **a) The time before the pie/face impact**
- **b) The distance of the Troll to the cliff**
- **c) The maximum height of the pie**

$V_x = 65\cos 37 = 52 \text{ m/s}$

$V_y = 65 \sin 37 = 39 \text{ m/s}$

1 time to apex

$V_f = V_0 + at$

$t = \frac{V_0}{a} = \frac{-39}{-9.8} = 4\text{s}$

2 d to apex

$d = V_0 t + \frac{1}{2}at^2$

$= 39(4) + .5(-9.8)\,3^2$

$= 136.4\text{m}$

$d_T = 136.4\text{m} + 125\text{m} =$ b) $\boxed{261.4\text{m} = d_{max}}$

3 time to ground

$d = \cancel{V_0 t} + \frac{1}{2}at^2$

$t = 7.3\text{s}$ a) $\boxed{t_{total} = t_{up} + t_{down} = 11.3\text{s}}$

4 $d_x = V_x t$

$= 52(11.3)$

c) $\boxed{d_x = 587.6\text{m}}$

Ex. #10. !!CHALLENGE!! A baseball is struck from home-plate at an angle of 35° and 1m